

Entrepreneurial Overconfidence, Self-Financing, and Capital-Market Efficiency*

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Abstract

This paper studies the impact of entrepreneurial overconfidence on self-financing and capital-market efficiency. We generalize Rochet and Freixas (2008) model of competitive capital markets with adverse selection by assuming some entrepreneurs are overconfident and others underconfident. We show that the existence of biased entrepreneurs lowers the equilibrium fraction of projects' self-financing. We find that entrepreneurial overconfidence reduces capital-market efficiency when (i) no entrepreneur is underconfident or (ii) risk aversion is low and the ratio of overconfident to underconfident entrepreneurs is high. However, overconfidence improves capital-market efficiency when risk aversion is high and the ratio of overconfident to underconfident entrepreneurs is moderate.

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1 Introduction

Entrepreneurs undertake and carry out risky projects. To do that they look for investors willing to finance part of their projects and thereby share the projects' risk. If investors are risk-neutral and entrepreneurs are risk-averse, then first best efficiency requires that all entrepreneurs obtain 100 percent outside finance.

Capital market efficiency crucially depends on informational asymmetries between market participants. When entrepreneurs and investors know the value of each project the capital market is efficient (first-best solution).

In reality, information is asymmetric and entrepreneurs know the value of their own projects better than investors do. If entrepreneurs with good-quality projects have no ability to signal to investors the value of their projects, then in general the equilibrium outcome is inefficient. Akerlof (1970) shows that when informational asymmetries are substantial the market may even fail to exist (adverse selection).

Spence (1973) shows that signaling can reduce the welfare loss associated with asymmetric information. Leland and Pyle (1977) apply Spence's model to the problem of entrepreneurs seeking for outside finance for projects that only they know the value of. They consider that entrepreneurs can signal the quality of their projects by investing more or less of their wealth in them. In this way "good" projects can be separated from "bad" projects by their level of self-financing. However, signaling is costly because entrepreneurs are risk-averse and those with good projects need to retain a fraction of the risk of their projects instead of obtaining full-insurance on financial markets. Thus, signalling reduces the inefficiencies caused by asymmetric information but at a cost (second-best solution).

Entrepreneurs' accurate beliefs about their project's value are the cornerstone of the signalling mechanism. Entrepreneurs can truthfully signal the value of a project to investors only if they know its actual value. Yet, empirical literature provides overwhelming evidence that entrepreneurs hold biased beliefs about the quality of their projects. Most entrepreneurs overestimate their skills, are optimistic about the likelihood of achieving success, and the extent of success. A small number

of entrepreneurs underestimate their skills and are pessimistic about their future prospects.

Cooper et al. (1988) find a large discrepancy between businesses' survival rate and entrepreneurs' perceptions of success. In their survey, 68 percent of American entrepreneurs thought that the odds of their business succeeding were better than for others in the same sector while only 5 percent thought they were worse. Additionally, 33 percent believed that their business would be successful for sure whereas only half of the businesses survive 5 years after foundation. Pinfold (2001) finds similar results in a survey on new business founders in New Zealand. Arabsheibani et al. (2000) compare expectations of future prosperity to actual outcomes using British panel data, and find that self-employed are more optimistic than employees. For example, 4.6 times as many self-employed people forecast an improvement of their prosperity but experienced deterioration as forecast a deterioration but experienced an improvement. For the employees the ratio was 2.9.¹ Also suggestive of optimism is the evidence of Hamilton (2000) and Moskowitz and Vissing-Jorgensen (2002) that the expected financial returns to self-employment fall well below those in paid employment.

In this paper we study the impact of entrepreneurial overconfidence on self-financing and capital market efficiency. To do that we generalize Rochet and Freixas' (2008) model of competitive capital markets with adverse selection by assuming some entrepreneurs are mistaken about the quality of their projects. Overconfident (underconfident) entrepreneurs believe to have a good (bad) quality project, when, in fact, they have a bad (good) quality project. Investors know the fractions of good-quality projects, overconfident and underconfident entrepreneurs, and observe self-financing decisions.

We start by showing that if the fraction of biased entrepreneurs is not too high,

¹Gentry and Hubbard (2000), Hurst and Lusardi (2004), Puri and Robinson (2007), Friedman (2007), and others show that overconfident individuals are more likely to become entrepreneurs. Busenitz and Barney (1997), Lowe and Ziedonis (2006) find that entrepreneurs are more overconfident than managers.

then there exists separating equilibria where entrepreneurs who perceive to have a bad-quality project do not self-finance their projects whereas those who perceive to have a good-quality project partially self-finance their projects.

Next, we show that the existence of biased entrepreneurs lowers the equity price gap, that is, the difference between the equity price of partially self-financed projects and non self-financed projects. The existence of overconfident entrepreneurs lowers the equity price of partially self-financed projects since investors know that a fraction of those projects is of bad-quality. In addition, the existence of underconfident entrepreneurs raises the equity price of non self-financed projects since investors know that a fraction of those projects is of good-quality.

We show that entrepreneurs' biases lower the equilibrium fraction of self-finance. The lower equity price gap makes self-financing less attractive to entrepreneurs who perceive to have a bad-quality project. As a consequence, entrepreneurs who perceive to have a good-quality project need to self-finance a lower fraction of the project to signal to investors they have a good-quality project in an incentive compatible manner than they would have to if all entrepreneurs were rational.

We proceed by analyzing the impact of entrepreneurs' biases on aggregate self-finance. Entrepreneurs' biases have three effects on aggregate self-finance. First, overconfident entrepreneurs partially self-finance their projects whereas if they were rational they would choose no self-finance. Second, underconfident entrepreneurs do not self-finance their projects whereas if they were rational they would choose partial self-finance. Third, unbiased entrepreneurs with good-quality projects need to self-finance a lower fraction of their projects to signal to investors they have a good-quality project than they would have to if all entrepreneurs were rational.

We show that entrepreneurial overconfidence raises aggregate self-finance if no entrepreneur is underconfident. However, if there exist some underconfident entrepreneurs in the economy and the ratio of overconfident to underconfident entrepreneurs is not too high, then the first effect is dominated by the last two effects and aggregate self-finance is lower with biased entrepreneurs than with rational ones.

Finally, we analyze the impact of entrepreneurial overconfidence on capital market efficiency. We find that entrepreneurial overconfidence lowers capital market efficiency if no entrepreneur is underconfident. However, matters are not so straightforward when there exist some underconfident entrepreneurs in the economy. In this case, if entrepreneurs' risk aversion is sufficiently high and the ratio of overconfident to underconfident entrepreneurs is moderate, then capital market efficiency is higher with biased entrepreneurs than with rational ones. The intuition behind this result is as follows.

The existence of biased entrepreneurs implies that unbiased entrepreneurs with high-quality projects need a lower fraction of self-finance to signal their type to investors. Hence, unbiased entrepreneurs with high-quality projects bear a lower risk than in the rational case. If entrepreneurs' degree of risk aversion is sufficiently high, then the cost of self-financing is substantially lower. However, the existence of overconfident entrepreneurs lowers the equity price offered by investors to unbiased entrepreneurs with high-quality projects. When risk aversion is sufficiently high and the ratio of overconfident to underconfident entrepreneurs is moderate, the favorable impact on the cost of self-financing is greater than the unfavorable impact on the equity price and unbiased entrepreneurs with high-quality projects are better off.

This result is consistent with the theory of the second best. According to this theory, introducing a new distortion—entrepreneurs' biases—in an environment where another distortion is already present—private information about project's value—, may increase welfare.

This paper is part of the growing literature on the impact of behavioral biases on markets. DellaVigna and Malmendier (2004) and Gabaix and Laibson (2006) study market interactions between sophisticated firms and biased consumers. They find that in competitive markets, biased consumers may be indirectly exploited by sophisticated consumers. Sandroni and Squintani (2007) investigate the policy implications of overconfidence in insurance markets. They find that compulsory insurance fails to make all agents better off because it is detrimental to low-risk agents.

Our paper contributes to theoretical literature on the implications of entrepreneurial overconfidence for corporate-investment, corporate-finance, and capital-market efficiency.² A common finding in this literature is that overconfidence causes distortions in the economy as a whole. This result also holds in our model when (i) there are no underconfident entrepreneurs or (ii) risk aversion is low and the ratio of overconfident to underconfident entrepreneurs is high. In contrast, we find that entrepreneurial overconfidence can improve capital market efficiency if risk aversion is high and the ratio of overconfident to underconfident entrepreneurs is moderate.

2 The Model

Consider an economy consisting of a large number of entrepreneurs and investors. Each entrepreneur has a risky project, requiring a fixed investment of 1 and yielding random gross returns $\tilde{R} = 1 + \tilde{r}(\theta)$. Net returns $\tilde{r}(\theta)$ follow a normal distribution of mean θ and variance σ^2 . The variance σ^2 is the same for all projects, whereas θ can take two values: a low value θ_1 if the project's quality is low and a high value θ_2 if the project's quality is high, where $\theta_2 > \theta_1 > 0$. The project's net mean returns θ are known by the entrepreneur.

Entrepreneurs are risk averse and investors are risk neutral. Entrepreneurs have a constant absolute risk aversion (CARA) utility function $u(W) = -e^{-\rho W}$, where $\rho > 0$ is the entrepreneurs' coefficient of risk-aversion and W is entrepreneurs' final wealth. Entrepreneurs have enough initial wealth $W_0 > 1$ to self-finance their project. However, self-financing a risky project is costly to entrepreneurs because they are risk averse. Furthermore, self-finance is more costly for entrepreneurs with low quality projects because the net mean returns from self-finance are lower than those of entrepreneurs with high quality projects.

There are four types of entrepreneurs in the capital market: unbiased entrepreneurs with low quality projects, overconfident entrepreneurs with low quality projects, un-

²We review this literature in Section 5.

biased entrepreneurs with high quality projects, and underconfident entrepreneurs with high quality projects. Let $\pi = Pr(\theta = \theta_2) \in (0, 1)$ be the fraction of high quality projects, $\nu \in [0, \pi]$ the fraction of underconfident entrepreneurs, and $\kappa \in [0, 1 - \pi]$ the fraction of overconfident entrepreneurs. Investors cannot observe a project's net mean returns θ nor entrepreneurs' beliefs. Entrepreneurs and investors know ν , κ , and the distribution of θ .

Let γ be the fraction of a project's self-finance. Investors observe γ and consequently infer the project's quality. Investors then price each project according to the inferred quality. Entrepreneurs seek to maximize their perceived expected utility according to their perceptions.

In a separating equilibrium, self-finance choices are determined by entrepreneurs' beliefs about the net mean returns of their project. Entrepreneurs who believe to have a high quality project self-finance a fraction $\gamma > 0$ of their project. Since self-financing has no value other than signalling, entrepreneurs who believe to have a low quality project do not self-finance it ($\gamma = 0$).³

Among all entrepreneurs who choose a fraction of self-finance $\gamma > 0$, investors know that a fraction $\beta = \frac{\kappa}{\pi + \kappa - \nu}$ has low quality projects and a fraction $1 - \beta = \frac{\pi - \nu}{\pi + \kappa - \nu}$ has high quality projects. Among all entrepreneurs who do not self-finance the project, investors know that a fraction $\alpha = \frac{\nu}{1 - \pi - \kappa + \nu}$ has high quality projects and a fraction $1 - \alpha = \frac{1 - \pi - \kappa}{1 - \pi - \kappa + \nu}$ has low quality projects. Hence, investors' posterior belief that a project's quality is high after having observed γ is

$$\mu(\theta_2|\gamma) = \begin{cases} \alpha, & \text{if } \gamma = 0 \\ 1 - \beta, & \text{if } \gamma > 0 \end{cases}.$$

Competition in the capital market implies that investors break even. Therefore, the equity price offered to each group of entrepreneurs (those who choose $\gamma = 0$ and those who choose $\gamma > 0$) is a weighted average of low and high quality projects' net

³We assume that there is no pooling equilibrium where all entrepreneurs obtain 100 percent outside financing at equity price $P = (1 - \pi)\theta_1 + \pi\theta_2$. This is the case if $1 - \pi > \frac{\rho\sigma^2}{2\Delta\theta}$.

mean returns. Hence, investors' strategy is to offer the following equity price

$$P(\gamma) = E[\tilde{r}(\gamma)] = \begin{cases} \theta_1 + \alpha\Delta\theta, & \text{if } \gamma = 0 \\ \theta_2 - \beta\Delta\theta, & \text{if } \gamma > 0 \end{cases},$$

with $\Delta\theta \equiv \theta_2 - \theta_1$.

Define the equity price gap, ΔP , as the difference between the equity price of self-financed projects and the equity price of not self-financed projects, that is,

$$\Delta P = (\theta_2 - \beta\Delta\theta) - (\theta_1 + \alpha\Delta\theta) = (1 - \alpha - \beta)\Delta\theta. \quad (1)$$

We see from (1) that the equity price gap with biased entrepreneurs—when $\alpha + \beta > 0$ —is lower than the equity price gap with rational entrepreneurs— $\alpha + \beta = 0$. This happens because the presence of overconfident entrepreneurs lowers the equity price of self-financed projects and the presence of underconfident entrepreneurs raises the equity price of projects that are not self-financed.

In a separating equilibrium the price of equity for self-financed projects must be higher than the price of equity for projects that are not self-financed. This condition is satisfied if $\alpha + \beta < 1$, or, using the definitions of α and β

$$(1 - \pi)\nu + \pi\kappa < (1 - \pi)\pi \quad (2)$$

Condition (2) says that if the fractions of overconfident and underconfident entrepreneurs are sufficiently small, self-finance can serve as a signal of project's quality. When the fraction of biased entrepreneurs is too high, condition (2) is violated and separating equilibria may no longer exist. We assume from now on that condition (2) is satisfied.

In a separating equilibrium, underconfident entrepreneurs and unbiased entrepreneurs with a low quality project do not envy overconfident entrepreneurs and entrepreneurs with a high quality project, that is

$$u(W_0 + \theta_1 + \alpha\Delta\theta) \geq Eu[W_0 + (1 - \gamma)(\theta_2 - \beta\Delta\theta) + \gamma\tilde{r}(\theta_1)] \quad (3)$$

The left-hand side of (3) is the utility that underconfident entrepreneurs and unbiased entrepreneurs with a low quality project obtain from selling the entire project to investors. In this case entrepreneurs' final wealth is the sum of initial wealth W_0 and the equity price $\theta_1 + \alpha\Delta\theta$ paid by investors. The right-hand side of (3) is the utility that underconfident entrepreneurs and unbiased entrepreneurs with a low quality project expect to obtain if they partially self-finance their project. Such entrepreneurs expect to obtain as final wealth the sum of initial wealth W_0 , the revenue obtained from selling fraction $1 - \gamma$ of the project to investors at equity price $\theta_2 - \beta\Delta\theta$, and the revenue obtained from keeping fraction γ of the project with net random returns $\tilde{r}(\theta_1)$.

Furthermore, in a separating equilibrium overconfident entrepreneurs and entrepreneurs with a high quality project do not envy underconfident entrepreneurs and unbiased entrepreneurs with a low quality project, that is

$$Eu[W_0 + (1 - \gamma)(\theta_2 - \beta\Delta\theta) + \gamma\tilde{r}(\theta_2)] \geq u(W_0 + \theta_1 + \alpha\Delta\theta) \quad (4)$$

The left-hand side of (4) is the utility that overconfident entrepreneurs and entrepreneurs with a high quality project expect to obtain from partially self-financing their project. Such entrepreneurs expect to get as final wealth the sum of initial wealth W_0 , the revenue obtained from selling fraction $1 - \gamma$ of the project to investors at equity price $\theta_2 - \beta\Delta\theta$, and the revenue obtained from keeping fraction γ of the project with net random returns $\tilde{r}(\theta_2)$. The right-hand side of (4) is the utility that overconfident entrepreneurs and entrepreneurs with a high quality project obtain by selling the entire project to investors. In this case entrepreneurs' final wealth is the sum of initial wealth W_0 and the equity price $\theta_1 + \alpha\Delta\theta$ paid by investors.

We can rewrite the expected utilities in (3) and (4) as utilities noting that if $u(\tilde{x}) = -e^{-a\tilde{x}}$ with $\tilde{x} \sim N(\mu, \sigma^2)$, then $E[u(\tilde{x})] = -e^{-a\mu + \frac{a^2}{2}\sigma^2} = u(a\mu - \frac{a^2}{2}\sigma^2)$. Hence, in a separating equilibrium

$$u(W_0 + \theta_1 + \alpha\Delta\theta) \geq u\left[W_0 + (1 - \gamma)(\theta_2 - \beta\Delta\theta) + \gamma\theta_1 - \gamma^2\frac{\rho\sigma^2}{2}\right] \quad (5)$$

and

$$u \left[W_0 + (1 - \gamma)(\theta_2 - \beta\Delta\theta) + \gamma\theta_2 - \gamma^2 \frac{\rho\sigma^2}{2} \right] \geq u(W_0 + \theta_1 + \alpha\Delta\theta) \quad (6)$$

must be satisfied.

There exist a continuum of separating equilibria parametrized by a fraction of self-finance γ fulfilling (5) and (6). These equilibria can be Pareto-ranked. We focus our analysis on the least cost separating equilibrium—the one with the lowest fraction of self-finance—because it Pareto-dominates the other separating equilibria.

3 Self-Finance

We now analyze the impact of entrepreneurs' biases on self-finance. We start by showing that the equilibrium fraction of self-finance with biased entrepreneurs is lower than that with rational ones. We then provide conditions under which misperceptions raise or lower aggregate self-finance.

In the least cost separating equilibrium, (5) holds with equality while (6) is slack.⁴ Therefore, the equilibrium fraction of self-financing by overconfident entrepreneurs and unbiased entrepreneurs with high-quality projects, γ_B , is obtained by solving

$$W_0 + \theta_1 + \alpha\Delta\theta = W_0 + (1 - \gamma)(\theta_2 - \beta\Delta\theta) + \gamma\theta_1 - \gamma^2 \frac{\rho\sigma^2}{2} \quad (7)$$

with respect to γ . Solving (7) we have that γ_B is given by

$$\gamma_B = \frac{\Delta\theta}{\rho\sigma^2} \left[-(1 - \beta) + \sqrt{(1 - \beta)^2 + 2(1 - \alpha - \beta) \frac{\rho\sigma^2}{\Delta\theta}} \right],$$

When all entrepreneurs are rational, the least cost separating equilibrium fraction of self-financing by entrepreneurs with high-quality projects, γ_R , is given by

$$\gamma_R = \frac{\Delta\theta}{\rho\sigma^2} \left(-1 + \sqrt{1 + 2 \frac{\rho\sigma^2}{\Delta\theta}} \right),$$

⁴Condition (6) is satisfied for all γ less than or equal to $\bar{\gamma} = \frac{\Delta\theta}{\rho\sigma^2} \left[\beta + \sqrt{\beta^2 + 2(1 - \alpha - \beta) \frac{\rho\sigma^2}{\Delta\theta}} \right]$. In the least cost separating equilibrium condition (6) is slack since $\gamma_B < \bar{\gamma}$.

Our first result compares γ_B and γ_R .

Proposition 1: *The least cost separating equilibrium fraction of self-finance with biased entrepreneurs is lower than the least cost separating equilibrium fraction of self-finance with rational entrepreneurs.*

In the least cost separating equilibrium with rational entrepreneurs, those with low-quality projects are indifferent between not self-financing their projects and self-financing fraction γ_R of their projects. As we have seen, the presence of biased entrepreneurs lowers the equity price of self-financed projects and raises that of non self-financed projects. This makes self-financing less attractive to entrepreneurs who perceive to have a low-quality project. As a consequence, entrepreneurs who perceive to have a high-quality project need to self-finance a lower fraction of the project to signal to investors they have a high-quality project in an incentive compatible manner than they would have to if all entrepreneurs were rational.

We now study the impact of entrepreneurs' biases on aggregate self-finance. Aggregate self-finance with biased entrepreneurs is

$$S_B = (\pi - \nu + \kappa)\gamma_B, \quad (8)$$

that is, the sum of self-finance by proportion $\pi - \nu$ of unbiased entrepreneurs with high-quality projects and by proportion κ of overconfident entrepreneurs. Aggregate self-finance with rational entrepreneurs is equal to

$$S_R = \pi\gamma_R. \quad (9)$$

The change in aggregate self-finance is obtained by subtracting (9) from (8):

$$\begin{aligned} S_B - S_R &= (\pi - \nu + \kappa)\gamma_B - \pi\gamma_R \\ &= \kappa\gamma_B - \nu\gamma_R - (\pi - \nu)(\gamma_R - \gamma_B). \end{aligned} \quad (10)$$

We see from (10) that the existence of biased entrepreneurs has three effects on aggregate self-finance. First, overconfident entrepreneurs partially self-finance their

projects whereas if they were rational they would choose no self-finance. Second, underconfident entrepreneurs do not self-finance their projects whereas if they were rational they would choose partial self-finance. Third, unbiased entrepreneurs with good-quality projects self-finance a lower fraction of their projects than they would have to if all entrepreneurs were rational. The first effect increases aggregate self-finance and the second and third effects lower it.

Proposition 2 shows that entrepreneurial overconfidence raises aggregate self-financing when there are no underconfident entrepreneurs.

Proposition 2: *If some entrepreneurs are overconfident and there are no underconfident entrepreneurs, then aggregate self-finance is higher than when all entrepreneurs are rational.*

When there are no underconfident entrepreneurs, the increase in aggregate self-finance from overconfident entrepreneurs is higher than the reduction due to the lower fraction of self-financing by unbiased entrepreneurs with high-quality projects.

Can entrepreneurs' biases lower aggregate self-financing? Proposition 3 shows that the existence of underconfident entrepreneurs is a necessary (but not sufficient) condition for that to happen.

Proposition 3. *If some entrepreneurs are overconfident and others are underconfident, then aggregate self-financing is lower than when all entrepreneurs are rational if and only if*

$$\frac{\kappa}{\nu} < \frac{\Delta\theta}{\rho\sigma^2} \left(1 + 2\frac{\rho\sigma^2}{\Delta\theta} - \sqrt{1 + 2\frac{\rho\sigma^2}{\Delta\theta}} + \frac{\rho\sigma^2}{\Delta\theta} \frac{\pi}{1 - \pi} \right). \quad (11)$$

Condition (11) provides an upper bound for the ratio of overconfident to underconfident entrepreneurs. If this condition is satisfied, then the increase in aggregate self-finance due to the existence of overconfident entrepreneurs is less than the reduction in aggregate self-finance due to the existence of underconfident entrepreneurs and due to the lower fraction of self-financing by unbiased entrepreneurs with high-quality projects.

4 Capital Market Efficiency

In this section we characterize the impact of entrepreneurs' biases on capital market efficiency. To do that we compare welfare with biased entrepreneurs to that with rational entrepreneurs. Welfare is the weighted average of the expected utilities of each group of entrepreneurs because investors break even.

To evaluate the expected utility of a biased entrepreneur, we take the perspective of an outside observer who knows the actual projects' value. We denote $E[u(\theta_1|\theta_1)]$ the expected utility of an unbiased entrepreneur with a low-quality project, $E[u(\theta_1|\theta_2)]$ the expected utility of an underconfident entrepreneur, $E[u(\theta_2|\theta_1)]$ the expected utility of an overconfident entrepreneur, and $E[u(\theta_2|\theta_2)]$ the expected utility of an unbiased entrepreneur with a high-quality project. The expected utilities of entrepreneurs with low and high quality-projects when all entrepreneurs are rational are $E[u(\theta_1)]$ and $E[u(\theta_2)]$, respectively. Hence, welfare with biased entrepreneurs is

$$W_B = (1 - \pi - \kappa) E[u(\theta_1|\theta_1)] + \nu E[u(\theta_1|\theta_2)] + \kappa E[u(\theta_2|\theta_1)] + (\pi - \nu) E[u(\theta_2|\theta_2)].$$

Our first welfare result compares the expected utilities of each type of entrepreneur in the biased model to those of entrepreneurs with low and high quality projects in the rational model.

Proposition 4: *In the least cost separating equilibria of the models with biased and rational entrepreneurs:*

- (i) If $\alpha \geq 0$, then $E[u(\theta_1|\theta_1)] \geq E[u(\theta_1)]$;
- (ii) If $\alpha \geq \gamma_R$ then $E[u(\theta_1|\theta_2)] \geq E[u(\theta_2)]$;
- (iii) If $\alpha \geq 0$, then $E[u(\theta_2|\theta_1)] \geq E[u(\theta_1)]$;
- (iv) If $\alpha \geq \gamma_R - \gamma_B$, then $E[u(\theta_2|\theta_2)] \geq E[u(\theta_2)]$.

The first part of Proposition 4 tells us that unbiased entrepreneurs with low-quality projects attain at least the same expected utility as that attained by entrepreneurs with low-quality projects in the rational model. This happens because

biased beliefs raise the equity price of non self-financed projects as long as there are some underconfident entrepreneurs in the economy.

The second part of Proposition 4 tells us that, if the fraction of underconfident entrepreneurs is not too high, then underconfident entrepreneurs attain a lower expected utility than that attained by entrepreneurs with high-quality projects in the rational model. Biased beliefs have two opposite effects on the expected utility of underconfident entrepreneurs. First, underconfident entrepreneurs sell their project at a lower price with respect to the rational case because they do not self-finance their project. Second, since underconfident entrepreneurs do not self-finance the project, they bear a lower risk. The unfavorable equity price effect prevails over the favorable risk reduction effect when the number of underconfident entrepreneurs is not too high.

The third part of Proposition 4 tells us that overconfident entrepreneurs attain at least the same expected utility as that attained by entrepreneurs with low-quality projects in the rational model. Biased beliefs have two opposite effects on the expected utility of overconfident entrepreneurs. First, overconfident entrepreneurs benefit of a higher price of equity because they self-finance part of the project. Second, since overconfident entrepreneurs self-finance part of the project, they bear a higher risk. The favorable equity price effect prevails over the unfavorable risk increase effect when there exist some underconfident entrepreneurs.

The fourth part of Proposition 4 tells us that, if the fraction of underconfident entrepreneurs is sufficiently high, then unbiased entrepreneurs with high-quality projects attain at least the same expected utility as that attained by entrepreneurs with high-quality projects in the rational model. Biased beliefs have two opposite effects on the expected utility of unbiased entrepreneurs with high-quality projects. First, they lower the equity price of self-financed projects. Second, they lower the fraction of self-finance. Hence, unbiased entrepreneurs with high-quality projects face a lower equity price and bear a lower risk than in the rational case. The favorable risk reduction effect prevails over the unfavorable equity price effect when the

number of underconfident entrepreneurs is sufficiently high.

Our first result on the impact of entrepreneurial overconfidence on capital market efficiency follows directly from Proposition 4: entrepreneurial overconfidence reduces capital market efficiency when there are no underconfident entrepreneurs. This happens because: (i) the expected utility of unbiased entrepreneurs with low-quality projects is the same as that of entrepreneurs with low-quality projects in the rational model, (ii) the expected utility of overconfident entrepreneurs is the same as that of entrepreneurs with low-quality projects in the rational model, and (iii) the expected utility of unbiased entrepreneurs with high-quality projects is lower than that of entrepreneurs with high-quality projects in the rational model.

We now show that entrepreneurial overconfidence can improve capital market efficiency.

Proposition 5. *If some entrepreneurs are overconfident and others are underconfident, then welfare is higher than when all entrepreneurs are rational, if*

$$\rho > \frac{1}{\Delta\theta}, \quad (12)$$

and

$$\frac{\kappa}{\nu} < \frac{1}{\rho^2\sigma^2} \left[\left(1 + \sqrt{1 + \frac{2\rho\sigma^2}{\Delta\theta}} \right) \left(\rho^2\sigma^2 \frac{\pi}{1-\pi} + \frac{\rho\sigma^2}{\Delta\theta} + 1 \right) + \frac{\rho\sigma^2}{\Delta\theta} \right]. \quad (13)$$

Condition (12) provides a lower bound for coefficient of absolute risk aversion and condition (13) provides an upper bound for the ratio of overconfident to underconfident entrepreneurs. When these two conditions are satisfied, welfare with biased entrepreneurs is higher than with rational entrepreneurs.

Proposition 5 tells us the existence of biased entrepreneurs raises capital market efficiency when entrepreneurs' risk aversion is sufficiently high and the ratio of overconfident to underconfident entrepreneurs is not too high. The intuition behind this result is as follows.

The presence of biased entrepreneurs implies that unbiased entrepreneurs with high-quality projects need lower partial self-finance to signal their type to investors.

Hence, unbiased entrepreneurs with high-quality projects bear a lower risk than in the rational case. If entrepreneurs' degree of risk aversion is sufficiently high, then the cost of self-financing is substantially lower. However, if the ratio of overconfident to underconfident entrepreneurs is too high, then there is a sharp fall in the price of equity of unbiased entrepreneurs with high-quality projects which has an unfavorable impact on expected utility. Hence, if risk aversion is high and the ratio of overconfident to underconfident entrepreneurs is not too high, then the favorable impact of misperceptions on the need for partial self-financing dominates the unfavorable impact on the price of equity.

If the two inequalities of Proposition 5 go in the opposite direction, then entrepreneurs' biases reduce welfare.

5 Discussion

In this section we discuss several extensions of the model, our contribution to the literature, and policy implications.

5.1 Extensions

We can extend our analysis by assuming that low-quality projects yield negative net mean returns, that is, $\theta_1 < 0 < \theta_2$.

If entrepreneurs are rational and low-quality projects yield negative net mean returns, there exist separating equilibria where low-quality projects are not self-financed and high-quality projects are partially self-financed. Investors know that non self-financed projects have negative net mean returns and therefore they are not willing to offer a positive equity price for those projects. Therefore, entrepreneurs do not undertake low-quality projects.

If some entrepreneurs are biased, there exist separating equilibria where unbiased entrepreneurs with low-quality projects and underconfident entrepreneurs do not self-finance their projects and where unbiased entrepreneurs with high-quality projects

and overconfident entrepreneurs partially self-finance their projects.

If the net mean returns of low-quality projects are slightly negative, that is, $-\alpha\theta_2/(1-\alpha) < \theta_1 < 0$, then the expected returns of non self-financed projects are strictly positive. This implies that investors are willing to offer a positive equity price for non self-financed projects. Thus, biased beliefs raise the equity price for non self-financed projects and lower the equity price for partially self-financed projects and the qualitative nature of the results remains unchanged.

In contrast, if the net mean returns of low-quality projects are substantially negative, that is, $\theta_1 \leq -\alpha\theta_2/(1-\alpha)$, then the expected returns of non self-financed projects are strictly negative. This implies that investors are not willing to offer a positive equity price for non self-financed projects. Therefore, unbiased entrepreneurs with low-quality projects and underconfident entrepreneurs do not undertake their projects. Proposition 6 shows that when the net mean returns of low-quality projects are substantially negative, entrepreneurs' biases lower capital market efficiency.

Proposition 6. *If the net mean returns of low-quality projects are sufficiently negative, that is, $\theta_1 \leq -\frac{\alpha\theta_2}{1-\alpha}$, then welfare is lower with biased entrepreneurs than with rational entrepreneurs.*

The intuition behind this result is straightforward. The existence of underconfident entrepreneurs implies that some high-quality projects are not undertaken and this represents a welfare loss. Additionally, the existence of overconfident entrepreneurs implies that unbiased entrepreneurs with high-quality projects attain a lower utility than entrepreneurs with high-quality projects in the rational model. This happens because the unfavorable lower price of equity effect dominates the favorable lower fraction of self-finance effect.

In our model entrepreneurs are endowed with a low or a high-quality project and choose the level of self-finance. In reality, entrepreneurs effort is also an important factor for project returns. We consider an extension of the model where a project's net returns depend on its quality, the effort put in by the entrepreneur, and where quality and effort are complements. Entrepreneurs choose effort to maximize net

project returns minus cost of effort.

The assumption that quality and effort are complements implies that entrepreneurs who perceive to have high-quality projects put in higher effort than those who perceive to have low-quality projects. This increases the return of projects taken by overconfident entrepreneurs and reduces the return of projects taken by underconfident entrepreneurs by comparison with the model where effort is not a choice variable. As a consequence, the equity price gap in the model with endogenous effort and biased entrepreneurs is less than the equity price gap in the rational model but greater than the equity price gap in the model with exogenous effort and biased entrepreneurs. So, the main qualitative findings of the model with endogenous effort will be similar to those of the model where effort is not a choice variable.

However, the quantitative deviations from the rational model will be less pronounced since with an endogenous choice of effort, the equity price gap with biased entrepreneurs is closer to the equity price gap with rational entrepreneurs than in the model where effort is not a choice variable. One difference concerns the ex-post utility of overconfident entrepreneurs. In the model with exogenous effort, overconfident entrepreneurs are not worse off than if they were rational and all other entrepreneurs were also rational. In contrast, in the endogenous effort model, overconfident entrepreneurs exert an excessive effort and might end up worse off than if they were rational and all other entrepreneurs were also rational.

When some entrepreneurs are credit constrained, that is, they do not have enough initial wealth to self-finance their projects (i.e. $W_0 < 1$), the fraction of entrepreneurs self-financing part of their projects will change but the qualitative nature of our findings will remain the same.

It is also possible to generalize the model by assuming that high-quality projects are riskier than low-quality ones. Indeed, projects' net returns are usually proportional to the projects' risk. If that is the case we have $\tilde{r}(\theta_2) \sim N(\theta_2, \sigma_2^2)$ and $\tilde{r}(\theta_1) \sim N(\theta_1, \sigma_1^2)$, where $\theta_2 > \theta_1 > 0$ and $\sigma_2^2 > \sigma_1^2 > 0$. We find that the results change quantitatively but not qualitatively.

5.2 Contribution to Literature

We now explain how our paper contributes to the literature on capital-market signaling, corporate decisions and capital-market efficiency.

5.2.1 Capital-Market Signaling

Rochet and Freixas' (2008) model of competitive capital markets with adverse selection is based on Leland and Pyle (1977). In Rochet and Freixas (2008) project quality, θ , can take only two values, θ_1 or θ_2 , whereas in Leland and Pyle (1977) θ has a continuous distribution on a closed interval. In both models there is a unique stage in the capital raising process, only the project's net mean returns is entrepreneurs' private information, and entrepreneurs are perfectly informed about the project's quality.

Grinblatt and Hwang (1989) and Welch (1989) generalize Leland and Pyle's (1977) model by introducing several stages in the capital raising process to explain why firms underprice at the initial public offering (IPO). They assume that both the mean and the variance of the project's net returns are known by the entrepreneur but unknown to investors. Investors infer the project's net mean returns and its variance observing both the fraction of the equity retained by the entrepreneur and the offering price. They show that underpricing at the IPO can occur because a project's value is positively related to the degree of underpricing. Similarly, Welch (1989) finds that high-quality firms underprice at the IPO in order to obtain a higher price at a seasoned offering.

We extend Rochet and Freixas' (2008) model of competitive capital markets with adverse selection by relaxing the assumption that entrepreneurs are perfectly informed about the quality of the project. This is motivated by empirical evidence which shows that entrepreneurs are overconfident about their skills and optimistic about the outcome of their projects.

5.2.2 Corporate Decisions

We now review the literature that explores the implications of overconfidence for corporate decisions. We focus on mergers and acquisitions, corporate investment, corporate finance, dividend policy and innovation.

Roll (1986) shows that overconfidence can lead to value destroying mergers and acquisitions. Overconfident managers in bidding firms overpay for target companies because they overestimate their own ability to run them. Malmendier and Tate (2008) provide empirical evidence that overconfident CEOs are more likely to make lower-quality acquisitions when their firms are abundant in internal funds. Moreover, the odds of making an acquisition are 65 percent higher if the CEO is overconfident.

Goel and Thakor (2008) show that risk averse rational CEOs underinvest in projects relative to the optimal investment level of risk neutral shareholders. Instead, moderately overconfident CEOs invest more in projects than rational CEOs thereby mitigating the underinvestment problem. Similarly, Giat et al. (2010) demonstrate that entrepreneurs' overconfidence explains their large investments in their own enterprises. Overconfident entrepreneurs overestimate the expected returns of their projects. Hence, they massively self-finance their own enterprise. Moskowitz and Vissing-Jorgensen (2002) confirms this result empirically. In fact, they find that (on average) entrepreneurs overinvest in their own enterprise instead of diversifying risk investing in public equity (private equity puzzle). Malmendier and Tate (2005) show that overconfident managers overestimate the returns to their projects and therefore view external funds as extremely costly. Hence, they overinvest when they have abundant internal funds whereas they cut investment when they need external financing.

Heaton (2002) demonstrates that optimistic managers believe that capital markets undervalue their firms' risky securities, and may not undertake projects yielding positive returns if they must be externally financed. Hackbarth (2008) shows analytically that for the same reason overconfident managers choose higher debt levels and issue new debt more often than unbiased managers. Landier and Thesmar (2009)

show that short-term debt is more appropriate for optimistic entrepreneurs. This result is corroborated by the empirical findings of Dai and Ivanov (2010). Malmendier et al. (2011) find that managerial traits like overconfidence and formative early-life experiences help to explain variation in capital structure that cannot be explained by time-invariant firm differences in traditional capital structure determinants like tax deductibility of interest payments, bankruptcy costs, or asymmetric information between firms and capital market. They find that overconfident CEOs who overestimate their firms' future cash flows view external financing as overpriced, especially equity financing, use less external finance, and, conditional on accessing external capital, issue less equity than their peers. Malmendier et al. (2011) empirical findings are consistent with our model where overconfident entrepreneurs self-finance a larger share of their project that they would if they were rational because they believe their project will yield high returns.

Deshmukh et al. (2010) develop a model to study the impact of CEO overconfidence on dividend policy. They show analytically and empirically that the level of dividend payout is lower in firms managed by overconfident CEOs. In Ben-David et al. (2007) model, overconfident managers underestimate cash flow volatility and therefore use low discount rates to value these cash flows. They find that companies with overconfident managers are less likely to pay dividends. This result is confirmed by their empirical analysis and by the empirical study of Cordeiro (2009) on the impact of managers' overconfidence on dividend policy.

Galasso and Simcoe (2011) develop a career concern model where CEOs innovate to demonstrate their ability. They show analytically and empirically that overconfident CEOs are more likely to pursue innovation because they underestimate the projects' probability of failure. In line with these results, the empirical study of Hirshleifer et al. (2010) finds that overconfident CEOs undertake riskier projects, invest more in innovation, achieve a greater total quantity of innovation in innovative industries, and are more effective innovators. These findings are consistent with the implications of our results.

5.2.3 Capital-Market Efficiency

In de Meza and Southey (1996) banks have better information about project quality than entrepreneurs and some entrepreneurs are realists but others are optimists. They show that optimism can explain several well-known features of entrepreneurship. For example, the fact that purely self-financed entrepreneurs face higher business failure rates than debt-financed entrepreneurs can be explained by the fact that optimistic entrepreneurs prefer maximum self-finance of their projects and realistic ones prefer debt finance. They also show that the existence of optimistic entrepreneurs reduces credit-market efficiency.

In Manove and Padilla (1999) entrepreneurs have better information about project quality than banks and some entrepreneurs are realists but others are optimists. Entrepreneurs have no assets available for investment, so any investment must be financed by banks. Banks can use interest rates and collateral requirements to screen entrepreneurs with good projects from those with bad ones. Optimistic entrepreneurs underestimate the probability of default and so undervalue the cost of collateral. They find that optimists are willing to fully collateralize their loans and so collateral cannot be used to separate the optimists from the realists. Collateral serves to protect the banks against the errors of optimistic entrepreneurs, but competition between banks reduces interest rates, which further encourages optimists. As a consequence banks lend too much, thus reducing credit-market efficiency.

The result that entrepreneurial overconfidence causes distortions in the economy as a whole also holds in our model when (i) there are no underconfident entrepreneurs or (ii) risk aversion is low and the ratio of overconfident to underconfident entrepreneurs is high. In contrast, we find that entrepreneurial overconfidence can improve capital market efficiency if risk aversion is high and the ratio of overconfident to underconfident entrepreneurs is moderate.

5.3 Policy Implications

Our results have the following policy implications. When entrepreneurs' biases raise welfare no policy intervention is needed, i.e. the optimal policy is *laissez-faire*. This is the case when entrepreneurs are sufficiently risk averse and the ratio of overconfident to underconfident entrepreneurs is moderate (Proposition 5).

A policy intervention aimed at eliminating biases is necessary whenever these lower welfare. This is the case if there are some overconfident entrepreneurs but no underconfident entrepreneurs in the economy (corollary of Proposition 4), if entrepreneurs' risk aversion is low and the ratio of overconfident to underconfident entrepreneurs is high (Proposition 5), and when low-quality projects yield sufficiently negative net mean returns (Proposition 6).

Cooper et al. (1988) and Kahneman and Lovallo (1993) argue that organizational optimism is best alleviated by introducing an “outside” view, one capable of realizing all the reasons the “inside” view might be wrong. Outside experts can make the entrepreneur aware of the risks that the entrepreneur is taking by self-financing part of the project. External evaluation of the project by financial intermediaries (e.g. banks) may also help entrepreneurs to correctly assess the quality of their projects.

6 Conclusion

This paper generalizes Rochet and Freixas (2008) model of competitive capital markets with adverse selection by assuming some entrepreneurs are overconfident and others underconfident. Investors know the fractions of high-quality projects in the economy as well as the fractions of overconfident and underconfident entrepreneurs.

We find that entrepreneurial overconfidence lowers the equilibrium level of partial self-finance and the equity price gap—the gap between the equity price of partially self-financed and non self-financed projects. We also show that entrepreneurial overconfidence raises capital market efficiency if entrepreneurs' risk aversion is high and the ratio of overconfident to underconfident entrepreneurs is moderate.

7 References

- Akerlof, George A. 1970. The Market for Lemons: Quality Uncertainty and the Market Mechanism. *Quarterly Journal of Economics* 84(3): 488-500.
- Arabsheibani, G. D., De Meza, J., Maloney, J. and B. Pearson, 2000. And a Vision Appeared unto them of a Great Profit: Evidence of Self-deception among the Self-employed. *Economics Letters* 67 (1), 35-41.
- Busenitz, Lowell W., and Barney Jay B. 1997. Differences Between Entrepreneurs and Managers in Large Organizations: Biases and Heuristics in Strategic Decision-Making. *Journal of Business Venturing* 12(1): 9-30.
- Ben-David, I., Harvey, C. R. and Graham, J. R. 2007. Managerial Overconfidence and Corporate Policies. NBER Working Paper Series, Vol. w13711. Available at SSRN: <http://ssrn.com/abstract=1079308>.
- Cooper, A.C., Woo C.Y. , and Dunkelberg W.C. 1988. Entrepreneurs Perceived Chances for Success. *Journal of Business Venturing* 3(2): 97-108.
- Cordeiro, L. 2009. Managerial Overconfidence and Dividend Policy. Working Paper, London Business School.
- Dai, N., Ivanov, V. 2010. Entrepreneurial Optimism, Credit Availability, and Cost of Financing: Evidence from U.S. Small Businesses. SSRN eLibrary.
- De Meza, David and Southey, Clive, 1996. The Borrower's Curse: Optimism, Finance and Entrepreneurship. *Economic Journal* 106, 375–386.
- Della Vigna, Stefano and Ulrike Malmendier. 2004. Contract Design and Self-Control: Theory and Evidence. *Quarterly Journal of Economics* 119, no. 2:353-402.
- Deshmukh, S., Goel, A.M., and Howe, K.M. 2010. CEO Overconfidence and Dividend Policy. Available at SSRN: <http://ssrn.com/abstract=1496404>.
- Freixas, Xavier and Rochet Jean-Charles. 2008. Microeconomics of Banking. 2nd ed., MIT Press, Cambridge, Massachusetts.
- Friedman, Henry. 2007. Does Overconfidence Affect Entrepreneurial Investment? University of Pennsylvania, Wharton School.

- Gabaix, Xavier, and David Laibson. 2006. Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets. *Quarterly Journal of Economics* 121, no. 2:505-540.
- Galasso, A., Simcoe, T.S. 2011. CEO Overconfidence and Innovation. *Management Science* 57(8): 1469-1484.
- Gentry, William M., and Hubbard R. Glenn. 2000. Tax Policy and Entrepreneurial Entry. *American Economic Review* 90(2): 283-287.
- Giat, Yahel, Hackman, Steven T., Subramanian. 2010. Investment under Uncertainty, Heterogeneous Beliefs, and Agency Conflicts. *Review of Financial Studies* 23(4): 1360-1404.
- Goel, A.M., Thakor, A.V. 2008. Overconfidence, CEO Selection, and Corporate Governance. *The Journal of Finance* 63(6): 2737-2784.
- Grinblatt Mark, Hwang Chuan Yang. 1989. Signalling and the Pricing of New Issues. *The Journal of Finance*, 44(2): 393-420.
- Hackbarth, D. 2008. Managerial Traits and Capital Structure Decisions. *Journal of Financial and Quantitative Analysis* 43(4): 843-882.
- Hamilton, B.H., 2000. Does Entrepreneurship Pay? An Empirical Analysis of the Returns to Self-employment. *Journal of Political Economy* 108 (3): 604-631.
- Heaton, J.B. 2002. Managerial Optimism and Corporate Finance. *Financial Management* 31(2): 33 - 45.
- Hirshleifer, D. A., Teoh, S. H., and Low, A. 2010. Are Overconfident CEOs Better Innovators? Available at SSRN: <http://ssrn.com/abstract=1598021>.
- Hurst, Erik, and Lusardi Annamaria. 2004. Liquidity Constraints, Household Wealth, and Entrepreneurship. *Journal of Political Economy* 112(2): 319-347.
- Kahneman, D. and D. Lovallo, 1993. Timid Choices and Bold Forecasts: A Cognitive Perspective on Risk Taking. *Management Science* 39, 17-31.
- Landier, A., Thesmar, D. 2009. Financial Contracting with Optimistic Entrepreneurs. *The Review of Financial Studies* 22(1): 117-150.

- Leland, Hayne E and Pyle, David H. 1977. Informational Asymmetries, Financial Structure, and Financial Intermediation. *The Journal of Finance* 32(2):371-87.
- Lowe, Robert A., and Ziedonis Arvids A. 2006. Overoptimism and the Performance of Entrepreneurial Firms. *Management Science* 52(2): 173-186.
- Malmendier, U., Tate, G. 2005. CEO Overconfidence and Corporate Investment. *The Journal of Finance* 60(6): 2661-2700.
- Malmendier, U., Tate, G. 2008. Who Makes Acquisitions? CEO Overconfidence and the Market's Reaction. *Journal of Financial Economics* 89: 20-43.
- Malmendier, U., Tate, G., and Yan, J. 2011. Overconfidence and Early-Life Experiences: The Effect of Managerial Traits on Corporate Financial Policies. *The Journal of Finance* 66(5): 1687-1733.
- Manove, Michael and Padilla, A. Jorge, 1999. Banking (conservatively) with Optimists. *Rand Journal of Economics* 30(2): 324-350.
- Moskowitz, Tobias J. and Vissing-Jorgensen Annette. 2002. The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle? *American Economic Review* 92(4): 745-778.
- Pinfold, John F. 2001. The Expectations of New Business Founders. *Journal of Small Business Management* 39(3): 279-85.
- Puri, Manju, and Robinson David T. 2007. Optimism and Economic Choice. *Journal of Financial Economics* 86(1): 71-99.
- Roll, R. 1986. The Hubris Hypothesis of Corporate Takeovers. *The Journal of Business* 59(2): 197-216.
- Sandroni, Alvaro and Francesco Squintani. 2007. Overconfidence, Insurance, and Paternalism. *American Economic Review* 97(5):1994-2004.
- Spence, Michael. 1973. Job Market Signaling. *The Quarterly Journal of Economics* 87(3): 355-374.
- Welch, Ivo. 1989. Seasoned Offerings, Imitation Costs, and the Underpricing of Initial Public Offerings. *The Journal of Finance* 44(2): 421-449.

8 Appendix

Proof of Proposition 1: When some entrepreneurs are biased $\alpha + \beta \in (0, 1)$ and, from (7), the optimal fraction of self-finance is defined implicitly as:

$$\frac{\gamma_B^2}{1 - \gamma_B} \frac{\rho\sigma^2}{2\Delta\theta} + \frac{\alpha}{1 - \gamma_B} = 1 - \beta, \quad (14)$$

with $\gamma_B \in (0, 1)$. When all entrepreneurs are rational $\alpha = \beta = 0$ and the optimal fraction of self-finance is defined implicitly as

$$\frac{\gamma_R^2}{1 - \gamma_R} \frac{\rho\sigma^2}{2\Delta\theta} = 1, \quad (15)$$

with $\gamma_R \in (0, 1)$. We consider two cases: (i) $\beta > 0$ and $\alpha \geq 0$ and (ii) $\beta = 0$ and $\alpha > 0$.

(i) If $\beta > 0$, then the RHS of (14) is less than the RHS of (15). This implies that the LHS of (14) is less than the LHS of (15). That is,

$$\frac{\gamma_B^2}{1 - \gamma_B} \frac{\rho\sigma^2}{2\Delta\theta} + \frac{\alpha}{1 - \gamma_B} < \frac{\gamma_R^2}{1 - \gamma_R} \frac{\rho\sigma^2}{2\Delta\theta}. \quad (16)$$

If $\alpha \geq 0$, then the second term in the LHS of (16) is non-negative. Hence, (16) implies

$$\frac{\gamma_B^2}{1 - \gamma_B} < \frac{\gamma_R^2}{1 - \gamma_R}. \quad (17)$$

Since $\frac{x^2}{1-x}$ is strictly increasing in x for $x \in (0, 1)$, then (17) implies $\gamma_B < \gamma_R$.

(ii) If $\beta = 0$ then (16) holds as equality. If (16) holds as equality and $\alpha > 0$, then (17) is satisfied and $\gamma_B < \gamma_R$.

Q.E.D.

Proof of Proposition 2: We need to show that if $\alpha = 0$ and $\beta \in (0, 1)$, then $S_B > S_R$. If $\alpha = \nu = 0$, then $\kappa = \beta\pi/(1 - \beta)$ and $S_B = (\pi + \kappa)\gamma_B = \pi\gamma_B/(1 - \beta)$, where $\gamma_B = \frac{1}{\rho\lambda} \left[-(1 - \beta) + \sqrt{(1 - \beta)^2 + 2(1 - \beta)\rho\lambda} \right]$ with $\lambda \equiv \frac{\sigma^2}{\Delta\theta}$. We have that $S_R = \pi\gamma_R$ where $\gamma_R = \frac{1}{\rho\lambda} (-1 + \sqrt{1 + 2\rho\lambda})$. Hence, $S_B > S_R$ is equivalent to

$$\frac{\pi}{1 - \beta} \gamma_B > \pi\gamma_R$$

or

$$-(1 - \beta) + \sqrt{(1 - \beta)^2 + 2(1 - \beta)\rho\lambda} > -(1 - \beta) + (1 - \beta)\sqrt{1 + 2\rho\lambda}$$

or

$$(1 - \beta)^2 + 2(1 - \beta)\rho\lambda > (1 - \beta)^2 + 2(1 - \beta)^2\rho\lambda$$

or

$$(1 - \beta) > (1 - \beta)^2,$$

which is true since $\beta \in (0, 1)$.

Q.E.D.

Proof of Proposition 3: The amount of self-finance as a function of κ and ν is given by:

$$S_B(\kappa, \nu) = (\pi - \nu + \kappa)\gamma_B(\kappa, \nu). \quad (18)$$

A first-order Taylor series expansion of $S_B(\kappa, \nu)$ around $(0, 0)$ is given by:

$$S_B(\kappa, \nu) \approx S_B(0, 0) + \left. \frac{\partial S_B}{\partial \kappa} \right|_{(0,0)} \kappa + \left. \frac{\partial S_B}{\partial \nu} \right|_{(0,0)} \nu,$$

where $S_B(0, 0) = S_R$. We need to find out the two partial derivatives. From (18) we have

$$\begin{aligned} \left. \frac{\partial S_B}{\partial \kappa} \right|_{(0,0)} &= \gamma_B(\kappa, \nu)|_{(0,0)} + \frac{\partial S_B}{\partial \gamma_B} \left(\frac{\partial \gamma_B}{\partial \alpha} \frac{\partial \alpha}{\partial \kappa} + \frac{\partial \gamma_B}{\partial \beta} \frac{\partial \beta}{\partial \kappa} \right) \Big|_{(0,0)} \\ &= \gamma_R + (\pi - \nu + \kappa) \left[-\frac{1}{\rho\lambda} \left(\frac{1 + \rho\lambda}{\sqrt{1 + 2\rho\lambda}} - 1 \right) \right] \frac{1}{\pi} \Big|_{(0,0)} \\ &= \gamma_R - \frac{1}{\rho\lambda} \left(\frac{1 + \rho\lambda}{\sqrt{1 + 2\rho\lambda}} - 1 \right) \end{aligned}$$

where $\lambda = \frac{\sigma^2}{\Delta\theta}$.

From (18) we have

$$\begin{aligned}
\left. \frac{\partial S_B}{\partial \nu} \right|_{(0,0)} &= -\gamma_B(\kappa, \nu)|_{(0,0)} + \frac{\partial S_B}{\partial \gamma_B} \left(\frac{\partial \gamma_B}{\partial \alpha} \frac{\partial \alpha}{\partial \nu} + \frac{\partial \gamma_B}{\partial \beta} \frac{\partial \beta}{\partial \nu} \right) \Big|_{(0,0)} \\
&= -\gamma_R + (\pi - \nu + \kappa) \left[-\frac{1}{\sqrt{1+2\rho\lambda}} \right] \frac{1}{1-\pi} \Big|_{(0,0)} \\
&= -\gamma_R - \frac{1}{\sqrt{1+2\rho\lambda}} \frac{\pi}{1-\pi}.
\end{aligned}$$

Hence, we have

$$S_B(\kappa, \nu) - S_R \approx \left[\gamma_R - \frac{1}{\rho\lambda} \left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1 \right) \right] \kappa - \left(\gamma_R + \frac{1}{\sqrt{1+2\rho\lambda}} \frac{\pi}{1-\pi} \right) \nu.$$

The term inside square brackets is positive since $\rho\lambda > 0$ implies $\gamma_R > \frac{1}{\rho\lambda} \left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1 \right)$. Note: From the definition of γ_R we have that $\rho\lambda > 0$ implies $\frac{1}{\rho\lambda} (-1 + \sqrt{1+2\rho\lambda}) > \frac{1}{\rho\lambda} \left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1 \right)$. Thus, $S_B(\kappa, \nu) < S_R$ as long as

$$\left[\gamma_R - \frac{1}{\rho\lambda} \left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1 \right) \right] \kappa < \left(\gamma_R + \frac{1}{\sqrt{1+2\rho\lambda}} \frac{\pi}{1-\pi} \right) \nu$$

or

$$\frac{\kappa}{\nu} < \frac{\gamma_R + \frac{1}{\sqrt{1+2\rho\lambda}} \frac{\pi}{1-\pi}}{\gamma_R - \frac{1}{\rho\lambda} \left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1 \right)}. \quad (19)$$

Substituting $\gamma_R = \frac{1}{\rho\lambda} (-1 + \sqrt{1+2\rho\lambda})$ in (19), multiplying both sides of (19) by $\frac{\rho\lambda\sqrt{1+2\rho\lambda}}{\rho\lambda\sqrt{1+2\rho\lambda}}$, and simplifying terms we obtain

$$\frac{\kappa}{\nu} < \frac{1}{\rho\lambda} \left(1 + 2\rho\lambda - \sqrt{1+2\rho\lambda} + \rho\lambda \frac{\pi}{1-\pi} \right).$$

or, substituting $\lambda = \frac{\sigma^2}{\Delta\theta}$

$$\frac{\kappa}{\nu} < \frac{\Delta\theta}{\rho\sigma^2} \left(1 + 2\frac{\rho\sigma^2}{\Delta\theta} - \sqrt{1 + 2\frac{\rho\sigma^2}{\Delta\theta}} + \frac{\rho\sigma^2}{\Delta\theta} \frac{\pi}{1-\pi} \right).$$

Q.E.D.

Proof of Proposition 4: Unbiased entrepreneurs with low-quality projects and underconfident entrepreneurs sell their projects at equity price $\theta_1 + \alpha\Delta\theta$, therefore their utilities are given by

$$E[u(\theta_1|\theta_1)] = E[u(\theta_1|\theta_2)] = -e^{-\rho(W_0+\theta_1+\alpha\Delta\theta)}. \quad (20)$$

The expected utility of an overconfident entrepreneur is given by

$$E[u(\theta_2|\theta_1)] = -e^{-\rho\left[W_0+(1-\gamma_B)(\theta_2-\beta\Delta\theta)+\gamma_B\theta_1-\gamma_B^2\frac{\rho\sigma^2}{2}\right]},$$

which, using (7), can be simplified to

$$E[u(\theta_2|\theta_1)] = -e^{-\rho(W_0+\theta_1+\alpha\Delta\theta)}. \quad (21)$$

Finally, the expected utility of an unbiased entrepreneur with a high-quality project is

$$E[u(\theta_2|\theta_2)] = -e^{-\rho\left[W_0+(1-\gamma_B)(\theta_2-\beta\Delta\theta)+\gamma_B\theta_2-\gamma_B^2\frac{\rho\sigma^2}{2}\right]},$$

which, using the fact that $\gamma_B^2 = \frac{2\Delta\theta}{\rho\sigma^2}[(1-\beta)(1-\gamma_B)-\alpha]$, can be simplified to

$$E[u(\theta_2|\theta_2)] = -e^{-\rho(W_0+\theta_1+\alpha\Delta\theta+\gamma_B\Delta\theta)}. \quad (22)$$

The expected utility of an entrepreneur with a low-quality project when all entrepreneurs are rational is

$$E[u(\theta_1)] = -e^{-\rho(W_0+\theta_1)}, \quad (23)$$

and the expected utility of an entrepreneur with a high-quality project when all entrepreneurs are rational is

$$E[u(\theta_2)] = -e^{-\rho\left[W_0+(1-\gamma_R)\theta_2+\gamma_R\theta_2-\gamma_R^2\frac{\rho\sigma^2}{2}\right]},$$

which, using the fact that $\gamma_R^2 = \frac{2\Delta\theta}{\rho\sigma^2}(1-\gamma_R)$, can be simplified to

$$E[u(\theta_2)] = -e^{-\rho(W_0+\theta_1+\gamma_R\Delta\theta)}. \quad (24)$$

(i) From (20) and (23) we have

$$E[u(\theta_1|\theta_1)] - E[u(\theta_1)] = e^{-\rho(W_0+\theta_1)} (1 - e^{-\rho\alpha\Delta\theta}).$$

Therefore, if $\alpha \geq 0$, then $E[u(\theta_1|\theta_1)] \geq E[u(\theta_1)]$.

(ii) From (24) and (20) it follows that

$$E[u(\theta_1|\theta_2)] - E[u(\theta_2)] = e^{-\rho(W_0+\theta_1)} (e^{-\rho\gamma_R\Delta\theta} - e^{-\rho\alpha\Delta\theta}).$$

Therefore, if $\alpha \geq \gamma_R$ then $E[u(\theta_1|\theta_2)] \geq E[u(\theta_2)]$.

(iii) From (23) and (21) it follows that

$$E[u(\theta_2|\theta_1)] - E[u(\theta_1)] = e^{-\rho(W_0+\theta_1)} (1 - e^{-\rho\alpha\Delta\theta}).$$

Therefore, if $\alpha \geq 0$, then $E[u(\theta_2|\theta_1)] \geq E[u(\theta_1)]$.

(iv) From (24) and (22) we have

$$E[u(\theta_2|\theta_2)] - E[u(\theta_2)] = e^{-\rho(W_0+\theta_1)} (e^{-\rho\gamma_R\Delta\theta} - e^{-\rho(\gamma_B\Delta\theta+\alpha\Delta\theta)}).$$

Therefore, if $\alpha \geq \gamma_R - \gamma_B$, then $E[u(\theta_2|\theta_2)] \geq E[u(\theta_2)]$.

Q.E.D.

Proof of Proposition 5: Welfare with biased entrepreneurs is given by:

$$W_B = (1 - \pi - \kappa) E[u(\theta_1|\theta_1)] + \nu E[u(\theta_1|\theta_2)] + \kappa E[u(\theta_2|\theta_1)] + (\pi - \nu) E[u(\theta_2|\theta_2)].$$

Making use of (20), (21) and (22), we have

$$W_B = -e^{-\rho(W_0+\theta_1+\alpha\Delta\theta)} [(1 - \pi + \nu) + (\pi - \nu) e^{-\rho\gamma_B\Delta\theta}], \quad (25)$$

where

$$\gamma_B = \frac{1}{\rho\lambda} \left[-(1 - \beta) + \sqrt{(1 - \beta)^2 + 2(1 - \alpha - \beta)\rho\lambda} \right], \quad (26)$$

and

$$\alpha = \frac{\nu}{1 - \pi - \kappa + \nu}, \quad (27)$$

and

$$\beta = \frac{\kappa}{\pi + \kappa - \nu}. \quad (28)$$

Taking a first-order Taylor series expansion of $W_B(\kappa, \nu)$ around $(0, 0)$ we obtain:

$$W_B(\kappa, \nu) \approx W_B(0, 0) + \left. \frac{\partial W_B}{\partial \kappa} \right|_{(0,0)} \kappa + \left. \frac{\partial W_B}{\partial \nu} \right|_{(0,0)} \nu, \quad (29)$$

where $W_B(0, 0) = W_R$. We need to find out the two partial derivatives. From (25) we have

$$\left. \frac{\partial W_B}{\partial \kappa} \right|_{(0,0)} = \left. \frac{\partial W_B}{\partial \gamma_B} \left(\frac{\partial \gamma_B}{\partial \alpha} \frac{\partial \alpha}{\partial \kappa} + \frac{\partial \gamma_B}{\partial \beta} \frac{\partial \beta}{\partial \kappa} \right) \right|_{(0,0)} + \left. \frac{\partial W_B}{\partial \alpha} \frac{\partial \alpha}{\partial \kappa} \right|_{(0,0)}.$$

From (25) we have

$$\left. \frac{\partial W_B}{\partial \gamma_B} \right|_{(0,0)} = \rho \Delta \theta \pi e^{-\rho(W_0 + \theta_1)} e^{-\rho \gamma_R \Delta \theta}$$

From (27) we obtain

$$\left. \frac{\partial \alpha}{\partial \kappa} \right|_{(0,0)} = \left. \frac{\nu}{(1 - \pi - \kappa + \nu)^2} \right|_{(0,0)} = 0.$$

From (28) we have

$$\left. \frac{\partial \beta}{\partial \kappa} \right|_{(0,0)} = \left. \frac{\pi - \nu}{(\pi + \kappa - \nu)^2} \right|_{(0,0)} = \frac{\pi}{\pi^2} = \frac{1}{\pi}.$$

From (26) we obtain

$$\begin{aligned} \left. \frac{\partial \gamma_B}{\partial \beta} \right|_{(0,0)} &= \frac{1}{\rho \lambda} \left[1 + \frac{1}{2} \frac{-2(1 - \beta) - 2\rho \lambda}{\sqrt{(1 - \beta)^2 + 2(1 - \alpha - \beta)\rho \lambda}} \right] \Big|_{(0,0)} \\ &= \frac{1}{\rho \lambda} \left[1 - \frac{(1 - \beta) + \rho \lambda}{\sqrt{(1 - \beta)^2 + 2(1 - \alpha - \beta)\rho \lambda}} \right] \Big|_{(0,0)} \\ &= -\frac{1}{\rho \lambda} \left(\frac{1 + \rho \lambda}{\sqrt{1 + 2\rho \lambda}} - 1 \right). \end{aligned}$$

Hence

$$\left. \frac{\partial W_B}{\partial \kappa} \right|_{(0,0)} = -e^{-\rho(W_0 + \theta_1)} \frac{\Delta \theta}{\lambda} \left(\frac{1 + \rho \lambda}{\sqrt{1 + 2\rho \lambda}} - 1 \right) e^{-\rho \gamma_R \Delta \theta}. \quad (30)$$

From (25) we have

$$\begin{aligned} \left. \frac{\partial W_B}{\partial \nu} \right|_{(0,0)} &= -e^{-\rho(W_0+\theta_1)} (1 - e^{-\rho\gamma_R\Delta\theta}) \\ &\quad + \frac{\partial W_B}{\partial \gamma_B} \left(\frac{\partial \gamma_B}{\partial \alpha} \frac{\partial \alpha}{\partial \nu} + \frac{\partial \gamma_B}{\partial \beta} \frac{\partial \beta}{\partial \nu} \right) \Big|_{(0,0)} + \frac{\partial W_B}{\partial \alpha} \frac{\partial \alpha}{\partial \nu} \Big|_{(0,0)}. \end{aligned}$$

From (27) we obtain

$$\left. \frac{\partial \alpha}{\partial \nu} \right|_{(0,0)} = \frac{1 - \pi - \kappa}{(1 - \pi - \kappa + \nu)^2} \Big|_{(0,0)} = \frac{1 - \pi}{(1 - \pi)^2} = \frac{1}{1 - \pi}.$$

From (28) we have

$$\left. \frac{\partial \beta}{\partial \nu} \right|_{(0,0)} = \frac{\kappa}{(\pi + \kappa - \nu)^2} \Big|_{(0,0)} = 0.$$

From (26) we obtain

$$\left. \frac{\partial \gamma_B}{\partial \alpha} \right|_{(0,0)} = \frac{1}{\rho\lambda} \left[\frac{1}{2} \frac{-2\rho\lambda}{\sqrt{(1-\beta)^2 + 2(1-\alpha-\beta)\rho\lambda}} \right] \Big|_{(0,0)} = -\frac{1}{\sqrt{1+2\rho\lambda}}.$$

From (25) we have

$$\left. \frac{\partial W_B}{\partial \alpha} \right|_{(0,0)} = \rho\Delta\theta e^{-\rho(W_0+\theta_1)} (1 - \pi + \pi e^{-\rho\gamma_R\Delta\theta})$$

Therefore

$$\left. \frac{\partial W_B}{\partial \alpha} \frac{\partial \alpha}{\partial \nu} \right|_{(0,0)} = \rho\Delta\theta e^{-\rho(W_0+\theta_1)} \left(1 + \frac{\pi}{1 - \pi} e^{-\rho\gamma_R\Delta\theta} \right).$$

$$\begin{aligned} \left. \frac{\partial W_B}{\partial \nu} \right|_{(0,0)} &= -e^{-\rho(W_0+\theta_1)} (1 - e^{-\rho\gamma_R\Delta\theta}) \\ &\quad - e^{-\rho(W_0+\theta_1)} \rho\Delta\theta e^{-\rho\gamma_R\Delta\theta} \left(\frac{1}{\sqrt{1+2\rho\lambda}} \frac{\pi}{1 - \pi} \right) \\ &\quad + e^{-\rho(W_0+\theta_1)} \rho\Delta\theta \left(1 + \frac{\pi}{1 - \pi} e^{-\rho\gamma_R\Delta\theta} \right). \end{aligned}$$

Hence

$$\begin{aligned} \left. \frac{\partial W_B}{\partial \nu} \right|_{(0,0)} &= -e^{-\rho(W_0+\theta_1)} \left[1 - e^{-\rho\gamma_R\Delta\theta} - \rho\Delta\theta \right. \\ &\quad \left. - \rho\Delta\theta \left(1 - \frac{1}{\sqrt{1+2\rho\lambda}} \right) \frac{\pi}{1-\pi} e^{-\rho\gamma_R\Delta\theta} \right] \end{aligned} \quad (31)$$

Substituting (30) and (31) into (29) we obtain

$$\begin{aligned} W_B(\kappa, \nu) - W_R &\approx -e^{-\rho(W_0+\theta_1)} \left\{ \frac{\Delta\theta}{\lambda} \left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1 \right) e^{-\rho\gamma_R\Delta\theta} \kappa \right. \\ &\quad \left. + \left[1 - \rho\Delta\theta - e^{-\rho\gamma_R\Delta\theta} - \rho\Delta\theta \left(1 - \frac{1}{\sqrt{1+2\rho\lambda}} \right) \frac{\pi}{1-\pi} e^{-\rho\gamma_R\Delta\theta} \right] \nu \right\}. \end{aligned}$$

or,

$$\begin{aligned} W_B(\kappa, \nu) - W_R &\approx -e^{-\rho(W_0+\theta_1)} \left\{ (1 - \rho\Delta\theta)\nu + \left\{ \frac{\Delta\theta}{\lambda} \left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1 \right) \kappa \right. \right. \\ &\quad \left. \left. - \left[1 + \rho\Delta\theta \left(1 - \frac{1}{\sqrt{1+2\rho\lambda}} \right) \frac{\pi}{1-\pi} \right] \nu \right\} e^{-\rho\gamma_R\Delta\theta} \right\}. \end{aligned} \quad (32)$$

From (32) we obtain two sufficient conditions for $W_B(\kappa, \nu) > W_R$:

$$\rho > \frac{1}{\Delta\theta},$$

and

$$\frac{\kappa}{\nu} < \frac{\lambda}{\Delta\theta} \frac{\sqrt{1+2\rho\lambda} + (\sqrt{1+2\rho\lambda} - 1) \frac{\pi}{1-\pi} \rho\Delta\theta}{1 + \rho\lambda - \sqrt{1+2\rho\lambda}}. \quad (33)$$

Rewriting the RHS of (33)

$$RHS = \frac{\lambda}{\Delta\theta} \frac{\sqrt{1+2\rho\lambda} + (\sqrt{1+2\rho\lambda} - 1) \frac{\pi}{1-\pi} \rho\Delta\theta}{1 + \rho\lambda - \sqrt{1+2\rho\lambda}}.$$

Multiplying both sides by $\frac{1+\rho\lambda+\sqrt{1+2\rho\lambda}}{1+\rho\lambda+\sqrt{1+2\rho\lambda}}$ and simplifying the denominator we have

$$RHS = \frac{\lambda}{\Delta\theta} \frac{(1 + \rho\lambda + \sqrt{1+2\rho\lambda}) \left[(1 + \frac{\pi}{1-\pi} \rho\Delta\theta) \sqrt{1+2\rho\lambda} - \frac{\pi}{1-\pi} \rho\Delta\theta \right]}{\rho^2 \lambda^2},$$

or,

$$RHS = \frac{(1 + \rho\lambda + \sqrt{1 + 2\rho\lambda}) \left[(1 + \frac{\pi}{1-\pi}\rho\Delta\theta)\sqrt{1 + 2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta \right]}{\rho^2\Delta\theta\lambda}.$$

Multiplying terms in the numerator we obtain

$$RHS = \left[(1 + \rho\lambda) \left(1 + \frac{\pi}{1-\pi}\rho\Delta\theta \right) \sqrt{1 + 2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta (1 + \rho\lambda) \right. \\ \left. + (1 + \frac{\pi}{1-\pi}\rho\Delta\theta)(1 + 2\rho\lambda) - \frac{\pi}{1-\pi}\rho\Delta\theta\sqrt{1 + 2\rho\lambda} \right] \div (\rho^2\Delta\theta\lambda).$$

Multiplying $(1 + \rho\lambda) \left(1 + \frac{\pi}{1-\pi}\rho\Delta\theta \right) \sqrt{1 + 2\rho\lambda}$ in the numerator we have

$$RHS = \left[\left(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho\Delta\theta + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta \right) \sqrt{1 + 2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta (1 + \rho\lambda) \right. \\ \left. + (1 + \frac{\pi}{1-\pi}\rho\Delta\theta)(1 + 2\rho\lambda) - \frac{\pi}{1-\pi}\rho\Delta\theta\sqrt{1 + 2\rho\lambda} \right] \div (\rho^2\Delta\theta\lambda).$$

Cancelling out $+\frac{\pi}{1-\pi}\rho\Delta\theta\sqrt{1 + 2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta\sqrt{1 + 2\rho\lambda}$ in the nominator we get

$$RHS = \left[\left(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta \right) \sqrt{1 + 2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta (1 + \rho\lambda) \right. \\ \left. + (1 + \frac{\pi}{1-\pi}\rho\Delta\theta)(1 + 2\rho\lambda) \right] \div (\rho^2\Delta\theta\lambda).$$

Multiplying terms in the nominator we obtain

$$RHS = \left[\left(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta \right) \sqrt{1 + 2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta - \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta \right. \\ \left. + 1 + 2\rho\lambda + \frac{\pi}{1-\pi}\rho\Delta\theta + 2\frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta \right] \div (\rho^2\Delta\theta\lambda).$$

Simplifying terms in the nominator we have

$$RHS = \frac{(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta) \sqrt{1 + 2\rho\lambda} + 1 + 2\rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta}{\rho^2\Delta\theta\lambda}.$$

Noting that $1 + 2\rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta = (1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta) + \rho\lambda$ we obtain

$$RHS = \frac{(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta) \sqrt{1 + 2\rho\lambda} + (1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta) + \rho\lambda}{\rho^2\Delta\theta\lambda}.$$

Evidencing out $(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta)$ we get

$$RHS = \frac{(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta)(1 + \sqrt{1 + 2\rho\lambda}) + \rho\lambda}{\rho^2\Delta\theta\lambda}.$$

Finally, replacing λ by $\frac{\sigma^2}{\Delta\theta}$ we have

$$RHS = \frac{1}{\rho^2\sigma^2} \left[\left(1 + \sqrt{1 + \frac{2\rho\sigma^2}{\Delta\theta}} \right) \left(\rho^2\sigma^2 \frac{\pi}{1-\pi} + \frac{\rho\sigma^2}{\Delta\theta} + 1 \right) + \frac{\rho\sigma^2}{\Delta\theta} \right]$$

Q.E.D.

Proof Proposition 6: Let the net mean returns of low-quality projects θ_1 be negative, that is $\theta_1 < 0 < \theta_2$. When all entrepreneurs are rational the equity price is

$$P(\gamma) = E[\tilde{r}(\gamma)] = \begin{cases} 0, & \text{if } \gamma = 0 \\ \theta_2, & \text{if } \gamma > 0 \end{cases}.$$

Investors do not finance low-quality projects because they yield negative net mean returns. Therefore, entrepreneurs with low-quality projects do not undertake their projects. When some entrepreneurs are biased the equity price is

$$P(\gamma) = E[\tilde{r}(\gamma)] = \begin{cases} 0, & \text{if } \gamma = 0 \text{ and } \theta_1 + \alpha\Delta\theta \leq 0 \\ \theta_1 + \alpha\Delta\theta, & \text{if } \gamma = 0 \text{ and } \theta_1 + \alpha\Delta\theta > 0 \\ \theta_2 - \beta\Delta\theta, & \text{if } \gamma > 0 \end{cases},$$

with $\Delta\theta \equiv \theta_2 - \theta_1$. If $\theta_1 \leq -\frac{\alpha\theta_2}{1-\alpha}$, then investors are not willing to offer a positive equity price for non self-financed projects. So, unbiased entrepreneurs with low-quality projects and underconfident entrepreneurs do not undertake their projects. In this case, the condition for a separating equilibrium is

$$\theta_2 - \beta\Delta\theta > 0. \tag{34}$$

We assume from now on that (34) is satisfied. The incentive compatibility condition for unbiased entrepreneurs with low-quality projects and underconfident entrepreneurs is

$$u(W_0) \geq Eu[W_0 + (1 - \gamma_B)(\theta_2 - \beta\Delta\theta) + \gamma_B\tilde{r}(\theta_1)].$$

In the least cost separating equilibrium this inequality is binding. Hence, using the property of normal returns and the fact that utilities are strictly increasing in final wealth we have

$$W_0 = W_0 + (1 - \gamma_B)(\theta_2 - \beta\Delta\theta) + \gamma_B\theta_1 - \gamma_B^2 \frac{\rho\sigma^2}{2}, \quad (35)$$

Rearranging terms in (35) we obtain that γ_B is given by

$$\gamma_B^2 \frac{\rho\sigma^2}{2} + \Delta\theta\gamma_B = \theta_2 - \beta\Delta\theta(1 - \gamma_B). \quad (36)$$

Hence, when entrepreneurs are rational, $\alpha = \beta = 0$, γ_R is given by

$$\gamma_R^2 \frac{\rho\sigma^2}{2} + \Delta\theta\gamma_R = \theta_2. \quad (37)$$

It follows from (36) and (37) that $\beta \geq 0$ implies $\gamma_R \geq \gamma_B$. The expected utilities of an unbiased entrepreneur with a low-quality project and of an underconfident entrepreneur are the same and given by

$$E[u(\theta_1|\theta_1)] = E[u(\theta_1|\theta_2)] = -e^{-\rho W_0}, \quad (38)$$

The expected utility of an overconfident entrepreneur is given by

$$E[u(\theta_2|\theta_1)] = -e^{-\rho \left[W_0 + (1 - \gamma_B)(\theta_2 - \beta\Delta\theta) + \gamma_B\theta_1 - \gamma_B^2 \frac{\rho\sigma^2}{2} \right]},$$

which, using (35), can be simplified to

$$E[u(\theta_2|\theta_1)] = -e^{-\rho W_0}. \quad (39)$$

Finally, the expected utility of an unbiased entrepreneur with a high-quality project is

$$E[u(\theta_2|\theta_2)] = -e^{-\rho \left[W_0 + (1 - \gamma_B)(\theta_2 - \beta\Delta\theta) + \gamma_B\theta_2 - \gamma_B^2 \frac{\rho\sigma^2}{2} \right]},$$

which, using the fact that $\gamma_B^2 = \frac{2}{\rho\sigma^2}[\theta_2 - \Delta\theta(1 - \beta)\gamma_B - \beta\Delta\theta]$, can be simplified to

$$E[u(\theta_2|\theta_2)] = -e^{-\rho(W_0 + \Delta\theta\gamma_B)}. \quad (40)$$

The expected utility of an entrepreneur with a low-quality project when all entrepreneurs are rational is

$$E[u(\theta_1)] = -e^{-\rho W_0}, \quad (41)$$

and the expected utility of an entrepreneur with a high-quality project when all entrepreneurs are rational is

$$E[u(\theta_2)] = -e^{-\rho \left[W_0 + (1-\gamma_R)\theta_2 + \gamma_R\theta_2 - \gamma_R^2 \frac{\rho\sigma^2}{2} \right]},$$

which, using the fact that from (37) $\gamma_R^2 = \frac{2}{\rho\sigma^2}(\theta_2 - \Delta\theta\gamma_R)$, can be simplified to

$$E[u(\theta_2)] = -e^{-\rho(W_0 + \Delta\theta\gamma_R)}. \quad (42)$$

Welfare with biased entrepreneurs is given by

$$W_B = (1 - \pi - \kappa) E[u(\theta_1|\theta_1)] + \nu E[u(\theta_1|\theta_2)] + \kappa E[u(\theta_2|\theta_1)] + (\pi - \nu) E[u(\theta_2|\theta_2)].$$

Making use of (38), (39) and (40), we have

$$W_B = -e^{-\rho W_0} [(1 - \pi + \nu) + (\pi - \nu)e^{-\rho\Delta\theta\gamma_B}]$$

Welfare with rational entrepreneurs is given by

$$W_R = \pi E[u(\theta_2)] + (1 - \pi) E[u(\theta_1)]$$

Making use of (41) and (42) we have

$$W_R = -e^{-\rho W_0} [(1 - \pi) + \pi e^{-\rho\Delta\theta\gamma_R}]$$

Hence,

$$W_B - W_R = e^{-\rho W_0} [\pi (e^{-\rho\Delta\theta\gamma_R} - e^{-\rho\Delta\theta\gamma_B}) + \nu (e^{-\rho\Delta\theta\gamma_B} - 1)] \quad (43)$$

The term multiplying π in (43) is negative or zero because $\Delta\theta\gamma_R \geq \Delta\theta\gamma_B \Leftrightarrow \gamma_R \geq \gamma_B$ which is true. The term multiplying ν in (43) is negative because $\Delta\theta\gamma_B > 0 \Leftrightarrow \gamma_B > 0$ which is true. Hence, $W_B < W_R$ as long as $\nu > 0$ or $\kappa > 0$.

Q.E.D.